Chapter Six

Lines, Angles and Triangles

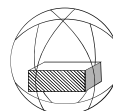
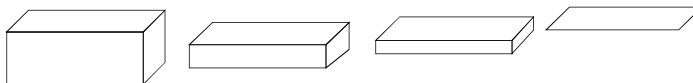
Geometry is an old branch of mathematics. The word 'geometry' comes from the Greek words 'geo', meaning the 'earth', and 'metrein', meaning 'to measure'. So, the word 'geometry' means 'the measurement of land.' Geometry appears to have originated from the need for measuring land in the age of agricultural based civilization. However, now a days geometry is not only used for measuring lands, rather knowledge of geometry is now indispensible for solving many complicated mathematical problems. The practice of geometry is evident in relics of ancient civilization. According to the historians, concepts and ideas of geometry were applied to the survey of lands about four thousand years ago in ancient Egypt. Signs of application of geometry are visible in different practical works of ancient Egypt, Babylon, India, China and the Incas civilisation. In the Indian subcontinent there were extensive usages of gometry in the Indus Valley civilisation. The excavations at Harappa and Mohenjo-Daro show the evidence of that there was a well planned city. For example, the roads were parallel to each other and there was a developed underground drainage system. Besides the shape of houses shows that the town dwellers were skilled in mensuration. In Vedic period in the construction of altars (or *vedis*) definite geometrical shapes and areas were maintained. Usually these were constituted with triangles, quadrilaterals and trapeziums.

But geometry as a systematic discipline evolved in the age of Greek civilization. A Greek mathematician, Thales is credited with the first geometrical proof. He proved logically that a circle is bisected by its diameter. Thales' pupil Pythagoras developed the theory of geometry to a great extent. About 300 BC Euclid, a Greek scholar, collected all the work and placed them in an orderly manner it in his famous treatise, 'Elements'. 'Elements' completed in thirteen chapters is the foundation of modern geometry for generations to come. In this chapter, we shall discuss logical geometry in accordance with Euclid.

At the end of this chapter, the students will be able to

- Describe the basic postulates of plane geometry
- Prove the theorems related to triangles
- Apply the theorems and corollaries related to triangles to solve problems.

6·1 Concepts of space, plane, line and point

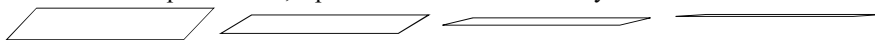
The space around us is limitless. It is occupied by different solids, small and large. By solids we mean the grains of sand, pin, pencil, paper, book, chair, table, brick, rock, house, mountain, the earth, planets and stars. The concepts

of geometry springs from the study of space occupied by solids and the shape, size, location and properties of the space.

A solid occupies space which is spread in three directions. This spread in three directions denotes the three dimensions (length, breadth and height) of the solid. Hence every solid is three-dimensional. For example, a brick or a box has three dimensions (length, breadth and height). A sphere also has three dimensions, although the dimensions are not distinctly visible.

The boundary of a solid denotes a surface, that is, every solid is bounded by one or more surfaces. For example, the six faces of a box represent six surfaces. The upper face of a sphere is also a surface. But the surfaces of a box and of a sphere are different. The first one is plane while the second is one curved.

Two-dimensional surface : A surface is two dimensional; it has only length and breadth and is said to have no thickness. Keeping the two dimension of a box unchanged, if the third dimension is gradually reduced to zero, we are left with a face or boundary of the box. In this way, we can get the idea of surface from a solid.

When two surfaces intersect, a line is formed. For example, two faces of a box meet at one side in a line. This line is a straight line. Again, if a lemon is cut by a knife, a curved line is formed on the plane of intersection of curved surface of the lemon.

One-dimensional line : A line is one-dimensional; it has only length and no breadth or thickness. If the width of a face of the box is gradually receded to zero, we are left with only line of the boundary. In this way, we can get the idea of line the from the idea of surface.

The intersection of two lines produces a point. That is, the place of intersection of two lines is denoted by a point. If the two edges of a box meet at a point. A point has no length, breadth and thickness. If the length of a line is gradually reduced to zero at last it ends in a point. Thus, a point is considered an entity of zero dimension.

6·2 Euclid's Axioms and Postulates

The discussion above about surface, line and point do not lead to any definition – they are merely description. This description refers to height, breadth and length, neither of which has been defined. We only can represent them intuitively. The definitions of point, line and surface which Euclid mentioned in the beginning of the first volume of his Elements' are incomplete from modern point of view. A few of **Euclid's axioms** are given below:

1. A **point** is that which has no part.
2. A line has no end point.

3. A **line** has only length, but no breath and height.

4. A **straight line** is a line which lies evenly with the points on itself.

5. A **surface** is that which has length and breadth only.

6. The edges of a surface are lines.

7. A **plane surface** is a surface which lies evenly with the straight lines on itself.

It is observed that in this description, part, length, width, evenly etc have been accepted without any definition. It is assumed that we have primary ideas about them. The ideas of point, straight line and plane surface have been imparted on this assumption. As a matter of fact, in any mathematical discussion one or more elementary ideas have to be taken granted. Euclid called them axioms. Some of the axioms given by Euclid are:

(1) Things which are eqal to the same thing, are eqal to one another.

(2) If eqals are added to eqals, the wholes are eqal.

(3) If eqals are subtracted from eqals, the remainders are eqal.

(4) Things which coincide with one another, are eqal to one another.

(5) The whole is greater than the part.

In modern geometry, we take a point, a line and a plane as undefined terms and some of their properties are also admitted to be true. These admitted properties are called geometric postulates. These postulates are chosen in such a way that they are consistent with real conception. The five postulates of Euclid are:

Postulate 1: *A straight line may be drawn from any one point to any other point.*

Postulate 2: *A terminated line can be produced indefinitely,*

Postulate 3: *A circle can be drawn with any centre and any radius.*

Postulate 4: *All right angles are equal to one another.*

Postulate 5: *If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.*

After Euclid stated his postulates and axioms, he used them to prove other results. Then using these results, he proved some more results by applying deductive reasoning. The statements that were proved are called **propositions or theorems**. Euclid is his ‘elements’ proved a total of 465 propositions in a logical chain. This is the foundation of modern geometry.

Note that there are some incompletion in Euclid’s first postulate. The drawing of a uniqe straight line passing through two distinct points has been ignored.

Postulate 5 is far more complex than any other postulate. On the other hand, Postulates 1 through 4 are so simple and obvious that these are taken as ‘self-evident truths’. However, it is not possible to prove them. So, these statements are accepted

without any proof. Since the fifth postulate is related to parallel lines, it will be discussed later.

6.3 Plane Geometry

It has been mentioned earlier that point, straight line and plane are three fundamental concepts of geometry. Although it is not possible to define them properly, based on our real life experience we have ideas about them. As a concrete geometrical conception space is regarded as a set of points and straight lines and planes are considered the subsets of this universal set.

Postulate 1. Space is a set of all points and plane and straight lines are the subsets of this set. From this postulate we observe that each of plane and straight line is a set and points are its elements. However, in geometrical description the notation of sets is usually avoided. For example, a point included in a straight line or plane is expressed by 'the point lies on the straight line or plane' or 'the straight line or plane passes through the point'. Similarly if a straight line is the subset of a plane, it is expressed by such sentences as 'the straight line lies on the plane, or 'the plane passes through the straight line'.

It is accepted as properties of straight line and plane that,

Postulate 2. For two different points there exists one and only one straight line, on which both the points lie.

Postulate 3. For three points which are not collinear, there exists one and only one plane, on which all the three points lie.

Postulate 4. A straight line passing through two different points on a plane lie completely in the plane.

Postulate 5. (a) Space contains more than one plane

(b) In each plane more than one straight lines lie.

(c) The points on a straight line and the real numbers can be related in such a way that every point on the line corresponds to a unique real number and conversely every real number corresponds to a unique point of the line.

Remark: The postulates from 1 to 5 are called incidence postulates.

The concept of distance is also an elementary concept. It is assumed that,

Postulate 6 : (a) Each pair of points (P, Q) determines a unique real number which is known as the ***distance*** between point P and Q and is denoted by PQ.

(b) If P and Q are different points, the number PQ is positive. Otherwise, $PQ=0$.

(c) The distance between P and Q and that between Q and P are the same, i.e. $PQ=QP$.

According to postulate 5(c) one to one correspondence can be established between the set of points in every straight line and the set of real numbers. In this connection, it is admitted that,

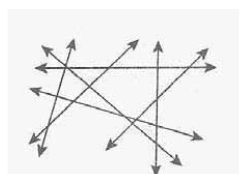
Postulate 7 : One-to-one correspondence can be established between the set of points in a straight line and the set of real numbers such that, for any points P and Q, $PQ = |a - b|$ where, the one-to-one correspondence associates points P and Q to real numbers a and b respectively.

If the correspondence stated in this postulate is made, the line is said to have been reduced to a number line. If P corresponds to a in the number line, P is called the graph point of P and a the coordinates of P. To convert a straight line into a number line the co-ordinates of two points are taken as 0 and 1 respectively. Thus a unit distance and the positive direction are fixed in the straight line. For this, it is also admitted that,

Postulate 8: Any straight line AB can be converted into a number line such that the coordinate of A is 0 and that of B is positive.

Remark: Postulate 6 is known as distance postulate and Postulate 7 as ruler postulate and Postulate 8 as ruler placement postulate.

Geometrical figures are drawn to make geometrical description clear. The model of a point is drawn by a thin dot by tip of a pencil or pen on a paper. The model of a straight line is constructed by drawing a line along a ruler. The arrows at ends of a line indicate that the line is extended both ways indefinitely. By postulate 2, two different points A and B define a unique straight line on which the two points lie. This line is called AB or BA line. By postulate 5(c) every such straight line contains infinite number of points.

According to postulate 5(a) more than one plane exist. There is infinite number of straight lines in every such plane. *The branch of geometry that deals with points, lines and different geometrical entities related to them, is known as plane Geometry.* In this textbook, plane geometry is the matter of our discussion. Hence, whenever something is not mentioned in particular, we will assume that all discussed points, lines etc lie in a plane.

Proof of Mathematical statements

In any mathematical theory different statements related to the theory are logically established on the basis of some elementary concepts, definitions and postulates. Such statements are generally known as propositions. In order to prove correctness of statements some methods of logic are applied. The methods are:

(a) Method of induction
(b) Method of deduction

Proof by contradiction

Philosopher Aristotle first introduced this method of logical proof. The basis of this method is:

- A property can not be accepted and rejected at the same time.
- The same object can not possess opposite properties.
- One can not think of anything which is contradictory to itself.
- If an object attains some property, that object can not unattain that property at the same time.

6·4 Geometrical proof

In geometry, special importance is attached to some propositions which are taken, as theorems and used successively in establishing other propositions. In geometrical proof different statements are explained with the help of figures. But the proof must be logical.

In describing geometrical propositions general or particular enunciation is used. The general enunciation is the description independent of the figure and the particular enunciation is the description based on the figure. If the general enunciation of a proposition is given, subject matter of the proposition is specified through particular enunciation. For this, necessary figure is to be drawn.

Generally, in proving the geometrical theorem the following steps should be followed :

(1) General enunciation.
(2) Figure and particular enunciation.
(3) Description of the necessary constructions and
(4) Description of the logical steps of the proof.

If a proposition is proved directly from the conclusion of a theorem, it is called a corollary of that theorem. Besides, proof of various propositions, proposals for construction of different figures are considered. These are known as constructions. By drawing figures related to problems, it is necessary to narrate the description of construction and its logical truth.

Exercise 6.1

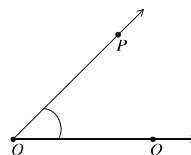
1. Give a concept of space, surface, line and point.
2. State Euclid’s five postulates.
3. State five postulates of incidence.
4. State the distance postulate.
5. State the ruler postulate.
6. Explain the number line.

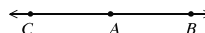
7. State the postulate of ruler placement.
8. Define intersecting straight line and parallel straight line.

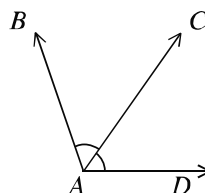
Line, Ray and Line Segment

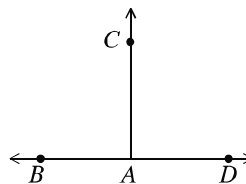
By postulates of plane geometry, every point of a straight line lies in a plane. Let AB be a line in a plane and C be a point on it. The point C is called internal to A and B if the points A, C and B are different points on a line and $AC + CB = AB$. The points A, C and B are also called collinear points. The set of points including A and B and all the internal points is known as the line segment AB. The points between A and B are called internal points.

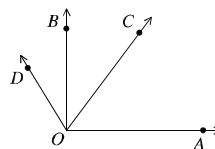
Angles

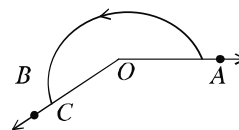
When two rays in a plane meet at a point, an angle is formed. The rays are known as the sides of the angle and the common point as vertex. In the figure, two rays OP and OQ make an angle $\angle POQ$ at their common point O. O is the vertex of the angle. The set of all points lying in the plane on the Q side of OP and P side of OQ is the known as the interior region of the $\angle POQ$. The set of all points not lying in the interior region or on any side of the angle is called exterior region of the angle.

Straight Angle

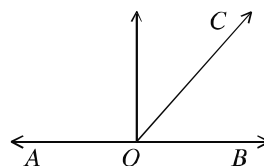
The angle made by two opposite rays at their common end point is a straight angle. In the adjacent figure, a ray AC is drawn from the end point A of the ray AB. Thus the rays AB and AC have formed an angle $\angle BAC$ at their common point A. $\angle BAC$ is a straight angle. The measurement of a right angle is 2 right angles or 180^0.

Adjacent Angle

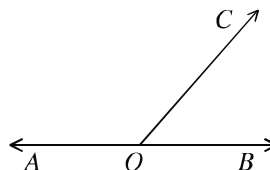
If two angles in a plane have the same vertex, a common side and the angles lie on opposite sides of the common side, each of the two angles is said to be an adjacent angle of the other. In the adjacent figure, the angles $\angle BAC$ and $\angle CAD$ have the same vertex A, a common side AC and are on opposite sides of AC. $\angle BAC$ and $\angle CAD$ are adjacent angles.

Perpendicular and Right Angle

The bisector of a straight angle is called perpendicular and each of th related adjacent angle a right angle. In the adjacent figure two angles $\angle BAC$ and $\angle CAD$ are produced at the point A of BD. The angles $\angle BAC$ and $\angle CAD$ are equal and lie on opposite sides of the common side AC. Each of the angles $\angle BAC$ and $\angle CAD$ is a right angle and the line segments BD and AC are mutually perpendicular.

Acute and Obtuse Angles

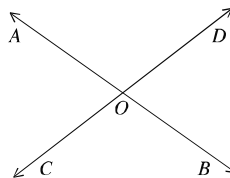
An angle which is less than a right angle is called an acute angle and an angle greater than one right angle but less than two right angles is an obtuse angle. In the figure, $\angle AOC$ is an acute angle and $\angle AOD$ is an obtuse angle

Reflex Angle

An angle which is greater than two right angles and less than four right angles is called a reflex angle. In the figure, $\angle AOC$ is a reflex angle.

Complementary Angles

If the sum of two angles is one right angle, the two angles are called complementary angles. In the adjacent figure, $\angle AOB$ is a right angle. The ray OC is in the inner side of the angle and makes two angles $\angle AOC$ and $\angle COB$. So taking together the measurement of these two angles is one right angle. The angles $\angle AOC$ and $\angle COB$ are complementary angles.

Supplementary Angles

If the sum of two angles is 2 right angles, two angles are called supplementary angles. The point O is an internal point of the line AB. OC is a ray which is different from the ray OA and ray OB. As a result two angles $\angle AOC$ and $\angle COB$ are formed. The measurement of this two angles is equal to the measurement of the straight angle $\angle AOB$ i.e., two right angles. The angles $\angle AOC$ and $\angle COB$ are supplementary angles.

Opposite Angles

Two angles are said to be the opposite angles if the sides of one are the opposite rays of the other.

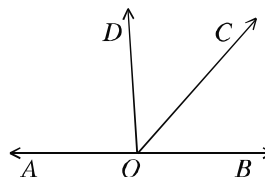
In the adjoining figure OA and OB are mutually opposite rays. So are the rays OC and OD. The angles $\angle AOC$ and $\angle BOD$ are a pair of opposite angles. Similarly, $\angle BOC$ and $\angle DOA$ are another pair of opposite angles. Therefore, two intersecting lines produce two pairs of opposite angles.

Theorem 1

The sum of the two adjacent angles which a ray makes with a straight line on its meeting point is equal to two right angles.

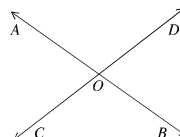
Let, the ray OC meets the straight line AB at O. As a result two adjacent angles $\angle AOC$ and $\angle COB$ are formed. Draw a perpendicular DO on AB.

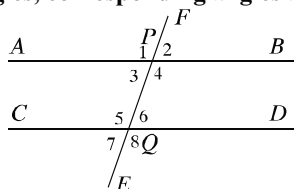
Sum of the adjacent two angles = $\angle AOC + \angle COB$ = $\angle AOD + \angle DOC + \angle COB$

$= \angle AOD + \angle DOB$

= 2 right angles. [Proved]

Theorem 2

When two straight lines intersect, the vertically opposite angles are equal.

Let AB and CD be two straight lines, which intersect at O. As a result the angles $\angle AOC$, $\angle COB$, $\angle BOD$, $\angle AOD$ are formed at O. $\angle AOC$ = opposite $\angle BOD$ and $\angle COB$ = opposite $\angle AOD$.

6·4 Parallel lines

Alternate angles, corresponding angles and interior angles of the traversal

In the figure, two straight lines AB and CD are cut by a straight line EF at P and Q. The straight line EF is a traversal of AB and CD. The traversal has made eight angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$ with the lines AB and CD. Among the angles

(a) $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$ are corresponding angles,

(b) $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$ are alternate angles,

(c) $\angle 4$, $\angle 6$ are interior angles on the right

(d) $\angle 3$, $\angle 5$ are interior angles on the left.

In a plane two straight lines may intersect or they are parallel. The lines intersect if there exists a point which is common to both lines. Otherwise, the lines are parallel. Note that two different straight lines may atmost have only one point in common.

The parallelism of two straight lines in a plane may be defined in three different ways:

(a) The two straight lines never intersect each other (even if extended to infinity)

(b) Every point on one line lies at equal smallest distance from the other.

(c) The corresponding angles made by a transversal of the pair of lines are equal.

According to definition (a) in a plane two straight lines are parallel, if they do not intersect each other. Two line segments taken as parts of the parallel lines are also parallel.

According to definition (b) the perpendicular distance of any point of one of the parallel lines from the other is always equal. Perpendicular distance is the length of the perpendicular from any point on one of the lines to the other. Conversely, if the perpendicular distances of two points on any of the lines to the other are equal, the lines are parallel. This perpendicular distance is known as the distance of the parallel lines.

The definition (c) is equivalent to the fifth postulate of Euclid. This definition is more useful in geometrical proof and constructions.

Observe that, through a point not on a line, a unique line parallel to it can be drawn.

Theorem 3

When a transversal cuts two parallel straight lines,

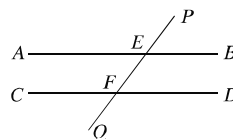
(a) the pair of alternate angles are equal.

(b) that pair of interior angles on the same side of the transversal are supplementary.

In the figure $AB \parallel CD$ and the transversal PQ intersects them at E and F respectively. Therefore,

(a) $\angle PEB$ = corresponding $\angle EFD$ [by definition]

(b) $\angle AEF$= alternate $\angle EFD$

(c) $\angle BEF + \angle EFD$ == 2 right angles.

Activity:

1. Using alternate definitions of parallel lines prove the theorems related to parallel straight lines.

Theorem 4

When a transversal cuts two straight lines, such that

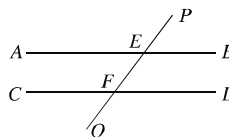
(a) pairs of corresponding angles are equal, or
(b) pairs of alternate interior angles are equal, or
(c) pairs of interior angles on the same side of the transversal are or equal to, the sum of two right angles the lines are parallel.

In the figure the line *PQ* intersects the straight lines *AB* and *CD* at *E* and *F* respectively and

(a) $\angle PEB$ = alternate $\angle EFD$

or, (b) $\angle AEF$ = Corresponding $\angle EFD$

or, (c) $\angle BEF + \angle EFD$ = 2 right angles.

Therefore, the straight lines *AB* and *CD* are parallel.

Corollary 1. The lines which are parallel to a given line are parallel to each other.

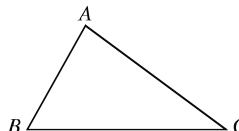
Exercise 6.2

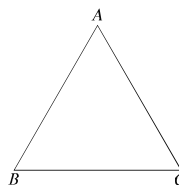
1. Define interior and exterior of an angle.
2. If there are three different points in a line, identify the angles in the figure.
3. Define adjacent angles and locate its sides.
4. Define with a figure of each: opposite angles, complementary angle, supplementary angle, right angle, acute and obtuse angle.

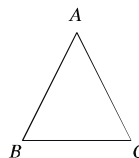
6.5 Triangles

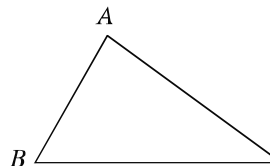
A triangle is a figure closed by three line segments. The line segments are known as sides of the triangle. The point common to any pair of sides is the vertex. The sides form angles at the vertices. A triangle has three sides and three angles. Triangles are classified by sides into three types: equilateral, isosceles and scalene. By angles triangles are also classified into three types: acute angled, right angled and obtuse angled.

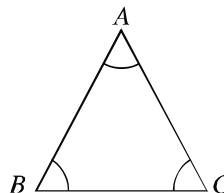
The sum of the lengths of three sides of the triangle is the perimeter. By triangle we also denote the region closed by the sides. The line segment drawn from a vertex to the mid-point of opposite side is known as the *median.* Again, the perpendicular distance from any vertex to the opposite side is the *height* of the triangle.

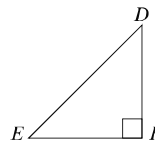
In the adjacent figure ABC is a triangle. A, B, C are three vertices. AB, BC, CA are three sides and $\angle BAC, \angle ABC, \angle BCA$ are three angles of the triangle. The sum of the measurement of AB, BC and CA is the perimeter of the triangle.

Equilateral Triangle

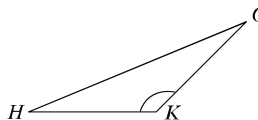
An equilateral triangle is a triangle of three equal sides. In the adjacent figure, triangle ABC is an equilateral triangle; because, $AB = BC = CA$ i.e., the lengths of three sides are equal.

Isosceles Triangle

An isosceles triangle is triangle with two equal sides. In the adjacent figure triangle ABC is an isosceles triangle; because $AB = AC \neq BC$ i.e., the lengths of only two sides are equal.

Scalene Triangle

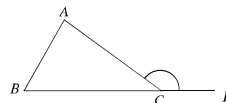
Sides of scalene triangle are unequal. Triangle ABC is a scalene triangle, since the lengths of its sides AB, BC, CA are unequal.

Acute Angled Triangle

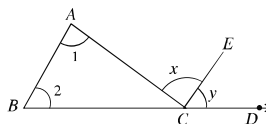
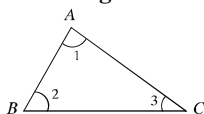
A triangle having all the three angles acute is acute angled triangle. In the triangle ABC each of the angles $\angle BAC$, $\angle ABC$ and $\angle BCA$ is acute i.e., the measurement of any angle is less than 90°. So ΔABC is acute angled.

Right Angled Triangle

A triangle with one of the angles right is a right angled triangle. In the figure, the $\angle DFE$ is a right angle; each of the two other angles $\angle DEF$ and $\angle EDF$ are acute. The triangle ΔDEF is a right angled triangle.

Obtuse angled triangle

A triangle having an angle obtuse is an obtuse angled triangle. In the figure, the $\angle GKH$ is an obtuse angle; the two other angles $\angle GHK$ and $\angle HGK$ are acute. ΔGHK is an obtuse angled triangle.

9.3 Interior and Exterior Angles

If a side of a triangle is produced, a new angle is formed. This angle is known as exterior angle. Except the angle adjacent to the exterior angle, the two others angles of the triangle are known as opposite interior angles.

In the adjacent figure, the side BC of ΔABC is produced to D. The angle $\angle ACD$ is an exterior angle of the triangle. $\angle ABC$, $\angle BAC$ and $\angle ACB$ are three interior angles. $\angle ACB$ is the adjacent interior angle of the exterior angle $\angle ACD$. Each of $\angle ABC$ and $\angle BAC$ is an opposite interior angle with respect to $\angle ACD$.

Theorem 5

The sum of the three angles of a triangle is equal to two right angles.

Let ABC be a triangle. In the triangle $\angle BAC + \angle ABC + \angle ACB = 2$ right angles.

Corollary 1: If a side of a triangle is produced then exterior angle so formed is equal to the sum of the two opposite interior angles.

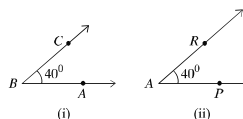
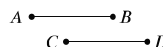
Corollary 2: If a side of a triangle is produced, the exterior angle so formed is greater than each of the two interior opposite angles.

Corollary 3: The acute angles of a right angled triangle are complementary to each other.

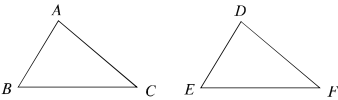
Activity :
1 Prove that if a side of a triangle is produced, the exterior angle so formed is greater than each of the two interior opposite angles.

Congruence of Sides and Angles

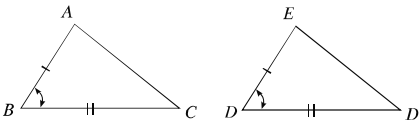
If two line segments have the same length, they are congruent. Conversely, if two line segments are congruent, they have the same length.

If the measurement of two angles is equal, the angles are congruent. Conversely, if two angles are congruent, their measurement is the same.

Congruence of Triangles

If a triangle when placed on another exactly covers the other, the triangles are congruent. The corresponding sides and angles of two congruent triangles are equal.

In the figure, ΔABC and ΔDEF are congruent. If two triangles ABC and DEF are congruent, by superposition of a copy of ABC on DEF we find that each covers the other completely. . Hence, the line segments as well angles are congruent. We would express this as $\Delta ABC \cong \Delta ABC$.

Theorem 6 (SAS criterion)

If two sides and the angle included between them of a triangle are equal to two corresponding sides and the angle included between them of another triangle, the triangles are congruent.

Let ABC and DEF be two triangles in which $AB = DE$, $AC = DF$ and the included $\angle BAC$ = the included $\angle EDF$. Then $\Delta ABC \cong \Delta DEF$.

Theorem 7

If two sides of a triangle are equal, the angles opposite the equal sides are also equal.

Suppose in the triangle ABC, $AB = AC$, then $\angle ABC = \angle ACB$.

Theorem 8

If two angles of a triangle are equal, the sides opposite the equal angles are also equal.

In the triangle ABC $\angle ABC = \angle ACB$. It is to be proved that $AB = AC$.

Proof

Steps	Justification
(1) If $AB = AC$ is not equal to AB, (i) $AB > AC$ or, (ii) $AB < AC$. Suppose, $AB > AC$. Cut from AB a part AD equal to AC. Now, the triangle ADC is an isosceles triangle. So, $\angle ADC = \angle ACD$	[The base angles of an isosceles triangles are equal]
In ΔDBC Exterior angle $\angle ADC > \angle ABC$ $\therefore \angle ACD > \angle ABC$ Therefore, $\angle ACB > \angle ABC$ But this is against the given condition.	[Exterior angle is greater than each of the interior opposite angles]

(2) Similarly, (ii) If $AB < AC$, it can be proved that $\angle ABC > \angle ACB$.

But this is also against the condition,

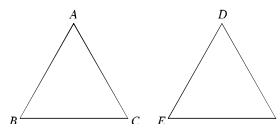
(3) So neither $AB > AC$ nor $AB < AC$

$\therefore$ $AB = AC$ (Proved)

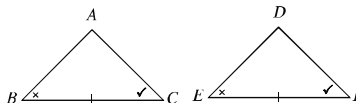
Theorem 9 (SSS criterion)

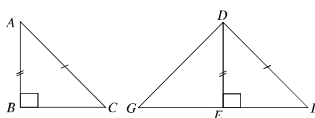
If the three sides of one triangle are equal to the three corresponding sides of another triangle, the triangles are congruent.

In ΔABC and ΔDEF, $AB = DE$, $AC = DF$ and $BC=EF$,

$\Delta ABC \cong \Delta ABC$.

Theorem 10 (ASA criterion)

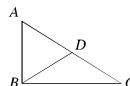
If two angles and the included side of a triangle are equal to two corresponding angles and the included side of another triangle, the triangles are congruent.

Let ABC and DEF be two triangles in which the $\angle B = \angle E$, $\angle C = \angle F$. and the side BC = the corresponding side EF, then the triangles are congruent, i.e. $\Delta ABC \cong \Delta DEF$.

Theorem 11 (HSA criterion)

If the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, the triangles are congruent.

Let ABC and DEF be two right angled triangles, in which the hypotenuse AC= hypotenuse DF and $AB = DE$, then $\Delta ABC \cong \Delta DEF$. .

Theorem 12

If one side of a triangle is greater than another, the angle opposite the greater side is greater than the angle opposite the lesser sides.

Let ABC be a triangle whose $AC > AB$, then $\angle ABC > \angle ACB$.

Theorem 13

If one angle of a triangle is greater than another, the side opposite the greater angle is greater than the side opposite the lesser.

Let ABC be a triangle in which $\angle ABC > \angle ACB$.
It is required to prove that, $AC > AB$.

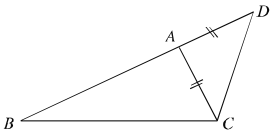
Proof :

Steps	Justification
(1) If the side AC is not greater than AB, (i) $AC = AB$ or, (ii) $AC < AB$	
(i) if $AC=AB$ $\angle ABC = \angle ACB$ which is against the supposition, since by supposition $\angle ABC > \angle ACB$	[The base angles of isosceles triangle are equal]
(ii) Again if $AC < AB$, $\angle ABC < \angle ACB$ But this is also against the supposition.	[The angle opposite to smaller side is smaller]
(2) Therefore, the side AC is neither equal to nor less than AB. $\therefore AC > AB$ (Proved).	

There is a relation between the sum or the differece of the lengths of two sides and the length of the third side of a triangle.

Theorem 14

The sum of the lengths of any two sides of a triangle is greater than the third side.

Let ABC be a triangle. Then any two of its sides are together greater than the third side. Let BC to be the greatest side. Then $AB+AC > BC$.

Corollary 1. The difference of the lengths of any two sides of a triangle is smaller than the third side.

Let ABC be a triangle. Then the difference of the lengths of any two of its sides is smaller than the length of third side i.e. $AB - AC < BC$.

Theorem 15

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and in length it is half.

Let ABC be a triangle and D and E are respectively mid-points of the AB and AC. It is required to prove that $DE \parallel BC$ and $DE = \frac{1}{2}BC$

Construction: Join D and E and extend to F so that $EF = DE$.

Proof :

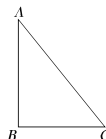
Steps	Justification
(1) Between ΔADE and ΔCEF, $AE = EC$	[given]
$DE = EF$	[by construction]
$\angle AED = \angle CEF$	[opposite angles]
$\Delta ADE \cong \Delta CEF$	[SAS theorem]
$\therefore \angle ADE = \angle EFC$ and $\angle DAE = \angle ECF$.	
$\therefore DF \parallel BC$ or, $DE \parallel BC$.	

(2) Again, $DF = BC$ or, $DE + EF = BC$ or, $DE + DE = BC$ or, $DE = \frac{1}{2}BC$

Theorem 16 (Pythagoras theorem)

In a right-angled triangle the square on the hypotenuse is equal to the sum of the squares of regions on the two other sides.

Let in the triangle ABC, $\angle ABC = 1$ right angle and AC is the hypotenuse.

Then $AC^2 = AB^2 + BC^2$,

Exercise 6.3

1 The lengths of three sides of a triang le are given below. In which case it is possible to draw a triangle?

(a) 5 cm, 6 cm and 7 cm (b) 3 cm, 4 cm and 7 cm

(c) 5 cm, 7 cm and **14** cm (a) 2 cm, 4 cm and 8 cm

2 Consider the following information:

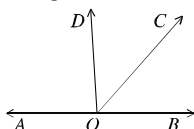
(i) A right angled triangle is a triangle with each of three angles right angle.

(ii) An acute angled triangle is a triangle with each of three angles acute.

(iii) A triangle with all sides equal is an equilateral triangle.

Which one of the following is correct?

(a) i and ii (b) i and iii (c) ii and iii (d) i, ii and iii

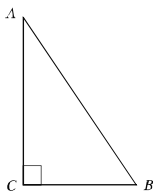
Use the figure below to answer questions 3 and 4.

3. Which one is a right angle?

   (a) $\angle BOC$ (b) $\angle BOD$ (c) $\angle COD$ (d) $\angle AOD$

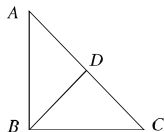
4. What is the angle complementary to BOC?

   (a) $\angle AOC$ (b) $\angle BOD$ (c) $\angle COD$ (d) $\angle AOD$

5. Prove that, the triangle formed by joining the middle points of the sides of an equilateral triangle is also equilateral.
6. Prove that, the three medians of an equilateral triangle are equal.
7. Prove that, the sum of any two exterior angles of a triangle is greater than two right angles.
8. D is a point inside a triangle ABC. Prove that, $AB + AC > BD + DC$.
9. If D is the middle point of the side BC of the triangle ABC, prove that, $AB + AC > 2AD$.
10. Prove that, the sum of the three medians of a triangle is less than its perimeter.
11. A is the vertex of an isosceles triangle ABC, and the side BA is produced to D such that $BA = AD$; prove that $\angle BCD$ = 1 right angle.
12. The bisectors of the angles $\angle B$ and $\angle C$ of a triangle ABC intersect at O. Prove that $\angle BOC = 90° + \frac{1}{2} \angle A$.
13. If the sides AB and AC of a triangle ABC are produced and the bisector of the exterior angles formed at B and C meet at O, prove that, $\angle BOC = 90° - \frac{1}{2} \angle A$.
14. In the adjoining figure, $\angle C$ = 1 right angle and $\angle B = 2 \angle A$.

    Prove that, $AB = 2BC$.

15. Prove that, the exterior angle so formed by producing any side of a triangle is equal to the sum of the interior opposite angles.

16. Prove that, the difference between any two sides of a triangle is less than the third.

17. In the adjoining figure, $\angle B =$ right angle and D is the middle point of the hypotenuse AC of the triangle ABC. Prove that. $BD = \frac{1}{2} AC$.

18. In the ΔABC, $AB > AC$ and the bisector AD of $\angle A$ intersects BC at D. Prove that $\angle ADB$ is an obtuse angle.

19. Show that, any point on the perpe ndicular bisector of a line segment is equidistant from the terminal points of that line segment.

20. In the rightangled triangle $\angle A =$ right angle and D is the mid point of BC.

   a. Draw a triangle ABC with given information.

   b. Prove that $AB + AC > 2AD$

   c. Prove that, $AD = \frac{1}{2} BC$